Fermilab

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CORRECTION OF SKEW QUADRUPOLE FIELD IN THE DOUBLER

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I.

As a measure of the total skew quadrupole field in a ring, it is convenient to define the quantity Q_s,

$$Q_s \equiv \oint ds\ B_s'/(B\rho)$$

where $(B\rho)$ is the rigidity of the beam and $B_s' \equiv \partial B_x/\partial x = -\partial B_y/\partial y$. For example, before any corrective measures were taken, we probably had $Q_s = +\ 1.1\times10^{-2}\ m^{-1}$ in the main ring at high field which was reduced to $(1.3 \sim 1.4)\times10^{-3}\ m^{-1}$ by rotating twelve horizontally defocusing quadrupoles.[1] The total roll angle is believed to be plus ("wall-side" down) 126 mrad. It has been pointed out[1] that, because of a peculiar nature of the main ring lattice, the coupling caused by the resonance $\nu_x - \nu_y = 0$ would not vanish even when Q_s is zero. Since the doubler lattice is similar but not identical (because of two high-beta insertions) to the main ring lattice, this point must be clarified again before the correction skew quadrupole system is fixed.

There are two sources of skew quadrupole field in the doubler:*

1. skew quadrupole field in dipoles. The field is expressed in terms of parameter a_1,

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* Skew quadrupole field arising from the coupling of closed-orbit distortions with the higher multipole fields is not considered here. Presumably, the closed orbit is well under control before one begins to worry about the effect of skew quadrupole field.

$$B_x(x) = B_o\, a_1\, x \qquad (\, y = 0 \,)$$

The present criterion for a_1 in dipoles is[2] $|a_1| < 2.5\times10^{-4}$/inch.

2. error in the magnetic axes of quadrupoles. When the rotation angle is θ_q, the skew quadrupole gradient is

$$B'_s = 2\theta_q |\partial B_y/\partial x|$$

where the normal quadrupole gradient $|\partial B_y/\partial x|$ for $\nu = 19.4$ is

$$|\partial B_y/\partial x| \text{ in kG/m} \simeq 0.759\times p(\text{in GeV/c}).$$

The angle θ_q is defined such that $\theta_q > 0$ when the wall-side is down (up) in horizontally focusing (defocusing) quadrupoles. The tentative criterion for θ_q, which is a combination of the measurement error and the alignment error, is[3]

$$(\theta_q)_{rms} = 3 \text{ mrad.}$$

There are reasons to believe that the present criteria for normal and skew quadrupoles in dipoles are unrealistically tight and a new criterion for the normal quadrupole component has been proposed.[4] It is suggested here that the new criteria for a_1 should be

660A to 4,000A

$(a_1)_{rms} < 3\times10^{-4}$/inch,

$|a_1| < 6\times10^{-4}$/inch,

$(a_1)_{av}$ for all dipoles $< 0.5\times10^{-4}$/inch.

The criteria for θ_q are

$(\theta_q)_{rms} < 3$ mrad,

$|\theta_q| < 6$ mrad,

$(\theta_q)_{av}$ for all quadrupoles < 0.5 mrad.

The purpose of this note is to study the consequence of these criteria on the necessary correction system.

II.

According to the design report issuned in May 1979,[5] the tentative plan is to install 18 to 24 correction skew quadrupoles with the maximum strength of $|B_s' \ell|$ = 60 kG each. At that time, it was still too early to make a more detailed plan on the locations and on the number of independent power supplies ("knobs").

Since it is rather unlikely that we would be forced to operate the doubler near the sum resonance $\nu_x + \nu_y = 39$, the coupling resonance with a major effect on the beam is the difference resonance $\nu_x - \nu_y = 0$. With the transverse emittance of the beam more or less the same in horizontal and vertical directions, this resonance by itself should not cause any problem* and this is indeed the case in the main ring. Why then should one bother to install a correction system which is never entirely a trivial undertaking in a superconducting ring? As an answer to this, one may cite the following:

1) A "clean" accelerator is almost always easier to operate than a "dirty" one. The presence of coupling is a real nuisance in many beam diagonostic measurements, a typical example being tune measurements.

2) Coupling of the horizontal and vertical closed orbits is definitely a source of trouble. The vertical dispersion is an example of this effect which may be harmful in the colliding mode.[6]

3) Possibly the most serious effect of coupling appears during the extraction. A relatively large radial excursion of the beam may increase the vertical oscillation amplitude and this may give rise to a beam loss of impossible magnitude. The problem has been studied by M. Harrison[7] with a computer simulation of the extraction process.

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* The transverse oscillation energy is transfered periodically from one direction to the other but there is no pumping of energy from the longitudinal direction.

He took the skew quadrupole in dipoles to be $(a_1)_{av} = 1\times10^{-4}$/inch and $(a_1)_{rms} = 3.6\times10^{-4}$/inch, Gaussian distribution.* Ninety correction skew quadrupoles were placed at horizontally defocussing quadrupole locations. He also studied cases with $(a_1)_{av} = 3\times10^{-4}$/inch but the results were not much different (except for a stronger correction field required) from the cases with $(a_1)_{av} = 1\times10^{-4}$/inch. With 90 correction elements all in series, the vertical amplitude can be suppressed to $\sim$ 3.5mm at β = 100m which he regards as the maximum tolerable amount for the extraction.

I believe two points should be further clarified in this calculation. 1) Is it really necessary to have as many as 90 elements? If not, what is the minimum number one should have for the correction system? 2) If random rolls are introduced in quadrupoles as another source of skew quadrupole field, can one still control the coupling with one group of correction elements? It should be emphasized here that the treatment of linear coupling given below would most likely break down when special extraction quadrupoles are on and one is essentially interested in the motion of "unstable" particles. Numerical methods such as the one used by Harrison seem to be the only reliable way to study the particle motion during extraction since the Hamiltonian is not integrable for such cases.

III.

In the usual approximation of disregarding all terms in the Hamiltonian except for the driving term of the resonance under consideration, the coupled motion for the resonances $\nu_x \pm \nu_y = N$ is characterized completely by the complex coupling parameter c_o:

$$c_o = (1/4\pi) \oint ds\ (\beta_x\beta_y)^{1/2} (B_s'/B\rho)\ e^{iF(s)},$$

$$F(s) \equiv (\psi_x(s) - \nu_x\theta) \pm (\psi_y(s) - \nu_y\theta) + N\theta,$$

* He also included skew sextupoles, skew octupoles and skew decapoles but their effects are not clear from his results.

$$\psi(s) \equiv \int_0^s ds/\beta \quad , \qquad \theta \equiv s/(\text{average radius of the ring}).$$

If the initial horizontal amplitude is A_o and the vertical amplitude is zero, the maximum vertical amplitude is

$$\text{max. } A_y = |c_o| / \left[(\Delta/2)^2 \mp |c_o|^2\right]^{1/2}, \ \Delta \equiv \nu_x \pm \nu_y - N.$$

The motion is unstable for the sum resonance $\nu_x + \nu_y = N$ when $|\Delta/2| < |c_o|$. (See ref. 8 for more on the linear coupling of betatron oscillations.)

For the resonance $\nu_x - \nu_y = 0$ which is of our primary concern, $F(s) \simeq \psi_x(s) - \psi_y(s)$ since $\nu_x \simeq \nu_y$. The angle F(s) is shown in Fig.1 where the origin of s and ψ is chosen to be at AØ and the tune is 19.4 in both directions. If the parameter c_o is regarded as a vector, any correction element placed after a normal station can produce a vector with the angle of 60^o only. This may not be too bad if all skew quadrupole fields to be compensated are in dipoles and quadrupoles of normal cells; the angle F(s) varies from 50^o to 70^o there (see Fig. 1a). However, there are 24 quadrupoles and ∿36 dipoles in the ring (see Figs 1b and 1c) where the angle F(s) is different from (50^o∿70^o). Any skew quadrupole field in these elements cannot be completely cancelled by correction elements installed at normal stations. The fact that $(\beta_x\beta_y)$ is larger at long straight sections than in the normal cells makes the situation even worse. One sees from Figs. 1c that the best locations for the second set of correction elements are A11, D11, A49 and D49 where the angle F(s) is ∿20^o, a value furthest possible in the ring from the (50^o∿70^o) range.* It is of course conceivable that we select dipoles with very small values of a_1 for the long straight sections and eliminate the necessity of having the second set of correction elements. However, it is not at all celar whether one can achieve and maintain very small roll angles for all 24 special quadrupoles.

It is not difficult to demonstrate the importance of correction

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* I am excluding the possibility of installing strong skew quadrupoles in the warm long straight sections.

elements at stations #11 and #49. Without any skew fields but the 24 special quadrupoles rolled by a certain amount θ_q, one cannot eliminate the coupling caused by the resonance $\nu_x - \nu_y = 0$ no matter how many correction elements installed at normal stations. The amount of coupling is of course a function of the roll angle θ_q and $\Delta \equiv \nu_x - \nu_y$.

IV.

Numerical tracing of the coupled transverse motion has been done for many cases with the following parameters:

A. skew quadrupole field in the ring

$(a_1)_{av} = 0.5\times10^{-4}$/inch,	$(\theta_q)_{av} = 0.5$ mrad
$(a_1)_{rms} = 3\times10^{-4}$/inch,	$(\theta_q)_{rms} = 3$ mrad
$\lvert a_1\rvert < 6\times10^{-4}$/inch,	$\lvert\theta_q\rvert < 6$ mrad.

B. correction elements

1. For $\nu_x - \nu_y = 0$:	set A	All and D11
	set B	two normal stations in each sector, all 12 in series
2. For $\nu_x + \nu_y = 39$:	set A	B18, C18, E18, F18
	set B	A43, B43, D43, E43

The arrangement here is antisymmetric and, within each set, stations are all equivalent; e. g., $B_s'(B18) = - B_s'(C18) = - B_s'(E18) = B_s'(F18)$.

total number of correction elements = 22,
number of independent power supplies = 4.

Note that the first sets do not contribute to the driving term of the sum resonance and the second sets have no effect on the driving term of the difference resonance. This of course is true only in the usual approximation of integrable Hamiltonians.

The average value assumed here for a_1 or θ_q may turn out to be too small or too large compared to the actual value in the ring. For this reason, it is useful to separate the effects of average skew fields. There is of course no contribution to the sum resonance (no odd harmonics) from the average skew field. For $\nu_x = \nu_y$,

$$c_o = 0.0541\ \bar{a}_1 + 0.0449\ \bar{\theta}_q + i[0.0908\ \bar{a}_1 + 0.0598\ \bar{\theta}_q],$$

where the average value $\bar{a}_1$ is in 10^{-4}/inch and $\bar{\theta}_q$ is in mrad. The corresponding strengths of correction elements are, for p = 1 TeV/c,

$$\begin{aligned} \text{set 1A: } (B_s'\ell) \text{ in kG} &= -10.4\ \bar{a}_1 - 36.2\ \bar{\theta}_q\ , \\ \text{set 1B: } \qquad &= -67.0\ \bar{a}_1 - 40.8\ \bar{\theta}_q . \end{aligned}$$

More than 1,500 samples of random distribution with the parameters specified on p. 6 have been used to find the necessary gradient strength for four sets of correction elements. In order to cover $\gtrsim$ 90% of the cases, we need

$$\begin{aligned} |B_s'\ell| \quad \text{for set 1A} &= 49 \text{ kG } (\times\ 2), \\ 1B &= 57 \text{ kG } (\times\ 12), \\ 2A &= 42 \text{ kG } (\times\ 4), \\ 2B &= 43 \text{ kG } (\times\ 4). \end{aligned}$$

The present design of skew quadrupoles specifies max. $|B_s'\ell|$ = 60 kG which is barely enogh for set 1B. If the average values assumed here for a_1 and θ_q are indeed true in the doubler, we may need more than twelve elements for set 1B.

The importance of set 1A (at A11 and D11) to compensate the skew quadrupole field arising from the angular misalignment of special quadrupoles can be seen from the following results:*

* For these, 300 samples have been used.

max. $|B_s'\ell|$ required for set 1A to cover $\gtrsim$ 90% of the cases

1. $(a_1)_{av} = 0.5\times10^{-4}$/inch, $(a_1)_{rms} = 3\times10^{-4}$/inch,

   $\theta_q = 0$ for all quadrupoles.

   max. $|B_s'\ell| = 14$ kG

2. $(\theta_q)_{av} = 0.5$ mrad, $(\theta_q)_{rms} = 3$ mrad,

   $a_1 = 0$ in all dipoles. max. $|B_s'\ell| = 42$ kG

3. $(a_1)_{av} = 0.5\times10^{-4}$/inch, $(a_1)_{rms} = 3\times10^{-4}$/inch,

   $(\theta_q)_{av} = 0.5$ mrad, $(\theta_q)_{rms} = 3$ mrad,

   No skew field in the 24 special quadrupoles. max. $|B_s'\ell| = 15$ kG

The actual amount of coupling is a function of $|c_o|$ and $|\Delta| \equiv |\nu_x \pm \nu_y - N|$. However, the expression given on p. 5 is valid only when the effect of one particular driving term is dominant in the particle motion. In general, it has been found for many different pairs of tunes that the agreement with numerical results is good within a factor of $\lesssim 2$ in predicting the maximum vertical amplitude for a given initial horizontal amplitude. With the present system of 22 correction elements and four independent power supplies, the best one can expect in controlling the coupled motion is between 10% and 20%. That is, when the initial horizontal amplitude is 10mm, one can control the growth of vertical oscillation to within $(1\sim2)$mm. This could be reduced to $\sim$10% if the number of elements in set 1B is increased from 12 to 40. Clearly more systematic studies of the relation between the number of correction elements and the maximum vertical oscillation amplitude are needed for the design of the system.

Finally, it is important to realize that all calculations reported here have been done for the ideal doubler lattice. Since the phase advance ψ and the betatron function β would be different in the actual ring because of the error in quadrupole field, normal stations would no longer be all equivalent. Furthermore, the stations 11 and 49 may totally lose their uniqueness in the ring. One should like to see this point thoroughly investigated and such studies must be done for cases with and without a correction system for the 39th harmonic component of normal quadrupole field.[4]

References

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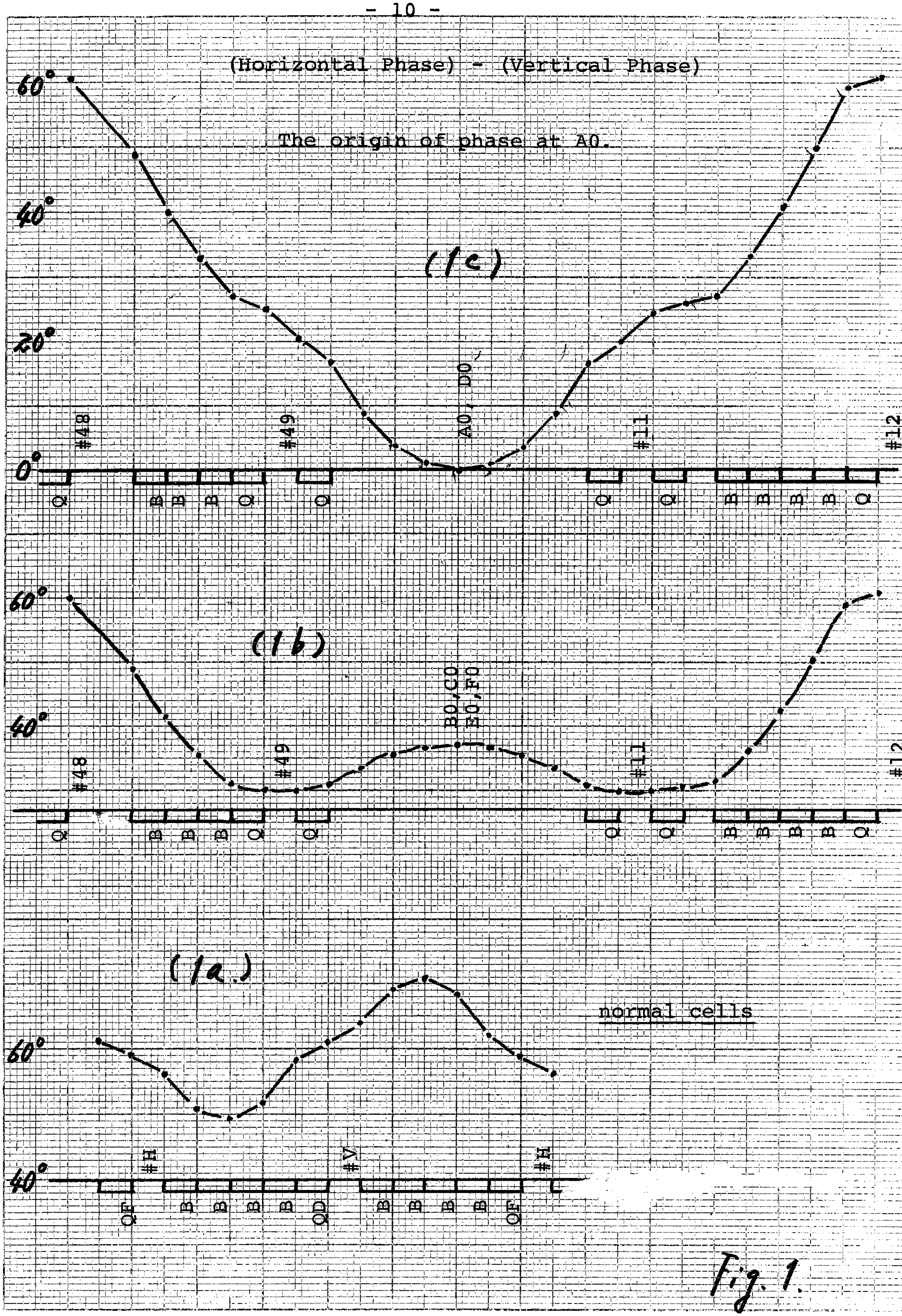
5. A Report on the Design of the Fermi National Accelerator Laboratory Superconducting Accelerator, May 1979.
6. S. Ohnuma, UPC No. 103, June 22, 1979.
7. M. Harrison, UPC No. 115, October 26, 1979.
8. EXP-85, September 14, 1977.


(Horizontal Phase) - (Vertical Phase)
The origin of phase at A0.
(1c)
60°
40°
20°
0°
#48
#49
A0 , D0
#11
#12
Q
B
B
B
Q
Q
Q
Q
B
B
B
B
Q
(1b)
60°
40°
B0 ,C0
E0 ,F0
#48
#49
#11
#12
Q
B
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B
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(1a.)
normal cells
60°
40°
#H
#V
#H
QF
B
B
B
B
QD
B
B
B
B
QF

Fig. 1.